

Talking about proportion: Fraction labels impact numerical interference in non-symbolic  
proportional reasoning

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**Author Notes**

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## Research Highlights

- Children are able to reason about proportional information well before they learn symbolic fractions, but also show predictable numerical biases.
- We find that the way proportional information is verbally labeled influences 6- and 7-year-olds' attention to number in a proportional reasoning task.
- Categorical labels that emphasized proportional similarities across equivalent proportions lead to less numerical interference than fraction labels emphasizing distinct numerical information.
- Thus, labels highlighting continuous features that are similar across equivalent proportions may benefit reasoning about non-symbolic proportion over traditional labels that highlight differences in number.

## Abstract

Across two experiments, we investigated how verbal labels impact the way young children attend to proportional information, well before the introduction of formal fraction education. Five- to seven-year-old children were introduced to equivalent non-symbolic proportions labeled in one of three ways: (1) a single, categorical label for multiple fractions (both  $\frac{3}{4}$  and  $\frac{6}{8}$  referred to as “blick”), (2) labels that focused on the numerator (e.g.,  $\frac{3}{4}$  labeled as “three blicks” (Experiment 1) or “three-fourths” (Experiment 2)), or (3) labels that had a complete part-whole structure (“three-out-of-four”). Children then completed measures of non-symbolic proportional reasoning that pitted whole-number information against proportional information for novel proportions. Across both experiments, children who heard the categorical labels were more likely to match non-symbolic displays based on proportion than children in any of the other conditions, who demonstrated higher levels of numerical interference. These findings suggest that fraction labels have the potential to shape children’s attention to proportional information even in the context of non-symbolic part-whole displays and for children who are not familiar with formal fraction symbols. We discuss these findings in terms of children’s developing understanding of proportional reasoning and its implications for fraction education.

Keywords: Proportion; Fractions; Labels; Whole Number Interference

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We regularly make informal judgments about proportional information in our environment, whether it be judgments of physical support (e.g., whether a book that is mostly, but not entirely, on a table is likely to fall off), chance and probability (e.g., if a bag of candy is mostly disliked candy, you wouldn't want to reach in and grab one at random), or interpreting commonly presented formal information (e.g., a pie chart during a work meeting). Although this is common practice throughout our lives, and even infants and young children show fairly sophisticated proportional reasoning ability (Denison & Xu, 2010; McCrink & Wynn, 2007; Sophian, Harley, & Martin, 1995; Spinillo, 2003), these proportion displays are not always straightforward. In particular, they are made difficult by the fact that proportion is a relation between two quantities and the magnitude of the two separate quantities can interfere with our attention to the *relation* between them (termed “numerical interference”; Boyer, Levine, & Huttenlocher, 2008; Hurst & Cordes, 2017; Jeong, Levine, & Huttenlocher, 2007; Ni & Zhou, 2005).

When proportion is presented discretely (i.e., with distinct, divided units that can be explicitly counted, like a bag of candy or a rectangle divided into quarters), this countable numerical information may be particularly salient and can interfere with our ability to focus on the relation between the two components. The interference between absolute number and relative proportion has been at the center of a substantial amount of work with symbolic (e.g., Alibali & Sidney, 2015; Durkin & Rittle-Johnson, 2015; Ni & Zhou, 2005) and non-symbolic representations (e.g., Boyer et al., 2008; Boyer & Levine, 2015; Fabbri, Caviola, Tang, Zorzi, & Butterworth, 2012; Hurst & Cordes, 2018; Jeong, et al., 2007). In particular, the availability of

countable, discrete quantities leads to systematic and predictable errors in the way children respond in proportion tasks. For example, children show success comparing and matching proportions when presented continuously (i.e., in the absence of countable numerical information); but when presented discretely, children respond in a way consistent with attending to the *number* of relevant pieces (i.e., the number of pieces in the “numerator”) rather than the *relation* between the number of relevant and total pieces (Boyer et al., 2008; Boyer & Levine, 2015; Hurst & Cordes, 2018; Jeong et al., 2007). Research has pointed to numerous factors that likely contribute to the prevalence of this numerical interference, such as teaching methods, previous experience, strategies, and individual differences in understanding fractions (e.g., Alibali & Sidney, 2015; Boyer & Levine, 2015; Empson, Junk, Dominguez, & Turner, 2006; Hurst & Cordes, 2018; Spinillo, 2002). In the current study, we investigate another factor that has been less explored – the words we use to talk about proportion: fraction labels.

Substantial research suggests that whole number words (e.g., “one”, “two”, “three”) are a key aspect of children’s early number knowledge (Baroody & Price, 1983; Condry & Spelke, 2008; Odic, LeCorre, & Halberda, 2015) and the *structure* of number words may be particularly important for how children learn basic number concepts (e.g., LeCorre & Carey, 2007; Fuson & Kwon, 1992; Miller, Smith, Zhu, & Zhang, 1995). For example, children who learn number words in languages that use base-10 number word systems (such as Korean and Chinese where 11 may be referred to as “ten and one” and 23 as “two tens and three”) learn to count faster and do better in number magnitude and arithmetic tasks than English speaking children (Fuson & Kwon, 1992; Laski & Yu, 2014; Miller & Stigler, 1987).

Although there is substantially less work investigating the impact of verbal fraction labels, there is some evidence that fraction labels do impact fraction learning (Paik & Mix, 2003;

Mix & Paik, 2008). The formal English system for fractions involves combining a whole-number label for the numerator with a number word variant for the denominator (e.g.,  $3/5$  is “three-fifths” and  $4/6$  is “four-sixths”). Furthermore, one of the unique aspects of fractions is that there are infinitely many distinct fractions representing the same magnitude (e.g.,  $3/4=6/8$ ).

Consequently, there are infinitely many labels for the same magnitude and equivalent fractions have different labels (e.g., three-quarters, six-eighths). Given all of this, our fraction label systems, which emphasizes whole-number information and does not convey information about the relation between the numerator and denominator, might be confusing for fraction learners. In support of this, some research suggests that cross-linguistic differences in fraction labels may be related to differences in cross-cultural performance (although, are not the whole story; Paik & Mix, 2003). Paik and Mix (2003) taught US children about symbolic fractions using the phrasing “four of five parts” and “of five parts, four” (based on a translation from Korean fraction labels) and found that this significantly helped children’s fraction reasoning over labels that just labeled the components (e.g., “four five”) and traditional US labels (“four-fifths”). Notably, these studies focused on how fraction labels impact learning fraction *symbols*.

In the current study, we investigate how the way we label fractions impacts reasoning about *non-symbolic* proportion, in the context of discrete visually presented proportions. To do so, we focused on a particularly challenging aspect of fraction learning that has both educational and theoretical implications: equivalent fractions. In particular, because fraction labels highlight the components that are different (the number of pieces) across equivalent fractions, while obscuring what is the same (the proportional magnitude), fraction labels may draw greater attention to number, promoting a numerical bias. As such, traditional fraction labels may impede children’s learning of equivalent fractions by focusing children’s attention on irrelevant parts

instead of overall proportion. In actuality, equivalent fractions are an equivalence class, or category, where the category level features are the same across exemplars (the proportion magnitude), but the features that are irrelevant to category membership are different (the specific parts/wholes). From this perspective, fraction labels function to highlight those features that are irrelevant to category membership. Interestingly, research on category learning suggests that using a single noun to refer to multiple category examples helps children attend to similarities across the exemplars and extract category-relevant features (e.g., Graham, Namy, Gentner, & Meagher, 2010; Plunkett, Hu, & Cohen, 2007). Thus, instead of referring to equivalent fractions each with a distinct label (e.g., “three-fourths”, “six-eighths”), children may benefit from using a single label for the entire equivalence class, allowing them to focus on the relational similarity across equivalent fractions and form a single proportional category (Singer-Freeman & Goswami, 2001).

In two experiments, we introduced 6.5-year-old children to non-symbolic representations of equivalent proportions ( $\frac{3}{4}$  and  $\frac{6}{8}$ ). Unlike much of the prior work (Boyer et al., 2008; Hurst & Cordes, 2018), we did not give children a cover story that emphasized the inherent importance of proportion (e.g., probabilities or recipe making), but instead used a cover story that emphasized category membership. This allowed us to investigate children’s spontaneous attention to numerical (i.e., non-categorical features) or proportional (i.e., categorical features) information on a subsequent task in the absence of the cover story. Importantly, however, we manipulated how the proportional stimuli were verbally labeled to explore how traditional and non-traditional labels may serve to highlight either the category membership or the individual features. Children were randomly assigned to different conditions that highlighted the category through the use of a single label (Categorical Condition in Experiments 1 and 2) or emphasized

numerical features, like the numerator (Numerator-Focused Condition in Experiment 1 and Traditional Condition in Experiment 2) or the part-whole nature of the proportion (Part-Whole Condition in Experiment 1 and Simplified Part-Whole Condition in Experiment 2). We hypothesized that using a single, consistent label would encourage children to treat equivalent proportions as belonging to the same category, increasing their attention to proportion and possibly preventing the numerical interference reported in previous studies. Conversely, those labels that do not highlight category membership and instead highlight differences across exemplars would not turn children's attention to proportion. We predicted that this would be particularly true for the labels emphasizing the numerator only, as they may function to turn children's attention *toward* number. There is mixed evidence impacting our predictions for the part-whole labels, which emphasize both the parts and the whole, but in a numerical way. On the one hand, prior work has found part-whole labels to be beneficial for *symbolic* fraction understanding (Paik & Mix, 2003; Mix & Paik, 2008), which may be because they help children extract proportional information. On the other hand, if the use of distinct number words across equivalent proportion is the primary difficulty, then part-whole labels may be just as likely to highlight numerical information (at the expense of proportional information) as labels that emphasize the numerator specifically.

## Experiment 1

### Method

**Participants.** Our sample consisted of 125 children ( $M_{\text{age}}=6.5$  years, range: 5.25 years to 7.8 years,  $n_{\text{female}}=55$ ), separated into three between-subject conditions: Numerator-Focused ( $n=41$ ,  $M_{\text{age}}=6.59$  years, range: 5.41 years to 7.8 years,  $n_{\text{female}}=18$ ), Part-Whole ( $n=41$ ,  $M_{\text{age}}=6.55$  years, range: 5.5 years to 7.8 years,  $n_{\text{female}}=18$ ), and Categorical ( $n=43$ ,  $M_{\text{age}}=6.46$



years, range: 5.25 years to 7.8 years,  $n_{\text{female}}=19$ ). An additional 17 children participated, but were excluded from analyses because of interference from parents or other children ( $n=2$ ) or experimenter error ( $n=15$ ).

Children participated in our campus laboratory or at local child-care programs and museums (the *Living Laboratory* at the Museum of Science, Boston, MA or the Acton Discovery Museum, Acton, MA) and received a small toy or sticker for participation. Parents or legal guardians provided written consent for each child and children over 7-years-old provided written assent. Demographic information was not systematically collected. However, based on other samples collected using similar recruitment methods in the lab and at local childcare centers and museums, we expect the sample to be predominantly white and educated, approximately as follows (these values are from different, but similar, samples): 72% White, 7% Asian, 2% Native Hawaiian/Pacific Islander, 2% Black or African American, and 17% mixed race; in addition, about 15% Hispanic. Lastly, most mothers were expected to have at least a Bachelor's degree (previous samples have shown values around 100%) and around 68% having a Master's degree or higher.

**Measures.** Participants completed five phases: (1) vocabulary assessment, (2) label training, (3) learning verification, (4) equivalence matching task, and (5) proportion magnitude comparison task. Children were randomly assigned to one of three conditions: *Categorical*, *Numerator-Focused*, and *Part-Whole*. The conditions only differed in the verbal information provided during the training phase, all other aspects of the tasks were identical. An experimenter administered all tasks and recorded the child's responses. The learning verification, equivalence matching, and comparison tasks were all presented on a 13-inch Mac Laptop using Xojo

programming software. Children were not given specific feedback about their accuracy but were given general encouragement (e.g., “You’re doing great! Keep going”).

**Vocabulary assessment.** A vocabulary assessment was included at the beginning of the session to ensure random assignment resulted in children across conditions displaying similar levels of vocabulary, since verbal information was the central manipulation across conditions. We used the standard protocol for administering the Woodcock-Johnson III Picture Vocabulary Test (Woodcock, McGrew, & Mather, 2001), in which children were shown pictures and asked for the name of the image. The experimenter started the test on the question recommended by the assessment based on the child’s grade and children were required to meet a basal criterion (first six questions administered correct or completed the first question) and testing ended once children answered six questions in a row incorrect. Children were live scored by the experimenter.

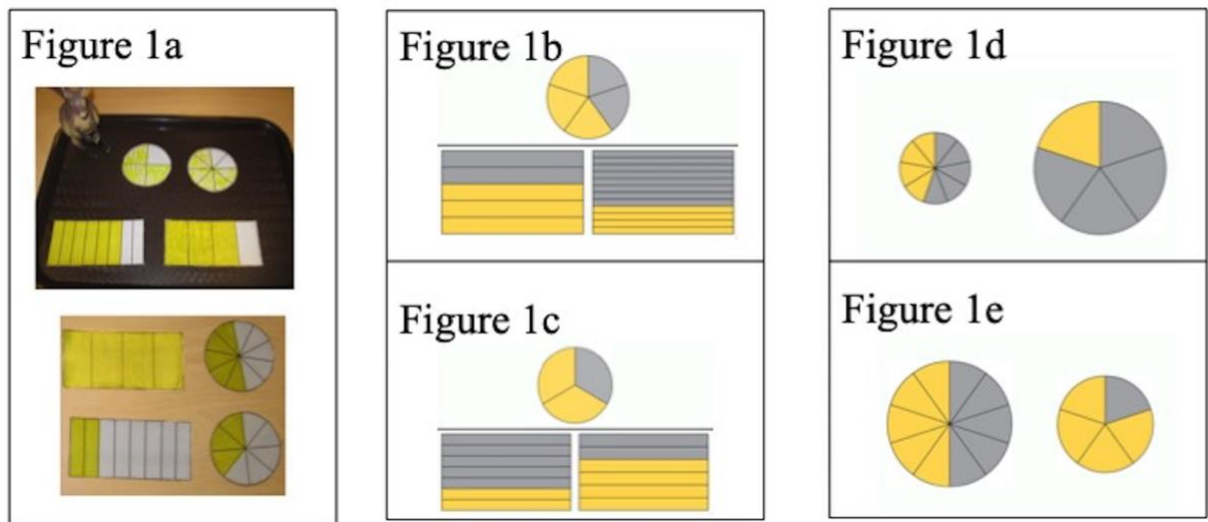
Some children did not have a useable vocabulary measure because they did not receive the vocabulary assessment (the test was added after starting data collection;  $n=13$ ), experimenter error during administration ( $n=6$ ), or because they were classified as an outlier ( $n=1$ , more than three standard deviations outside the mean). Data from 108 children were used in analyses involving vocabulary (Numerator-Focus:  $n=36$ ,  $M_{\text{age}}=6.67$  years, range: 5.66 years to 7.8 years,  $N_{\text{female}}=16$ ; Part-Whole:  $n=35$ ,  $M_{\text{age}}=6.6$  years, range: 5.5 years to 7.8 years,  $N_{\text{female}}=16$ ; Categorical:  $n=37$ ,  $M_{\text{age}}=6.44$  years, range: 5.25 years to 7.8 years,  $N_{\text{female}}=16$ ).

**Training phase.** In all conditions children were introduced to a toy animal character (e.g., a plastic kangaroo named Roo; the specific character varied, but for clarity, we refer to the character as “Roo” throughout) who “likes shapes that have just the right amount of color” (see Appendix for the full script). The game was introduced as a way to find out what amount of

color the character liked. First, children were shown an empty grey circle (8cm in diameter) divided into quarters (from the center, creating angular slices) and the experimenter colored in three adjacent pieces using a yellow marker saying: “I’m going to color [condition specific label]”. After coloring the pieces, the experimenter said: “See, this is called [condition specific label], Roo likes this one” and placed the shape on a tray in front of the character. The experimenter then brought out a rectangle (13.4cm long by 6.3cm tall, with vertical dividing lines along the shorter dimension into four equal pieces) that had three-of-four consecutive pieces already colored yellow and said “This is also [condition specific label], Roo likes this one too” and placed it on the tray. The experimenter then brought out two other pre-colored shapes (one circle and one rectangle, same dimensions as previously), with each one saying “But Roo won’t like this one” and placing the shape off to the side away from the other shapes, but still visible. This was then repeated with new shapes to demonstrate that Roo also likes shapes with  $\frac{6}{8}$  colored yellow. At the end, there were four shapes ( $\frac{3}{4}$  circle,  $\frac{3}{4}$  rectangle,  $\frac{6}{8}$  circle, and  $\frac{6}{8}$  rectangle) placed on the tray with the character, as exemplars of what Roo liked, and four counter-example shapes ( $\frac{3}{8}$  circle,  $\frac{4}{4}$  rectangle,  $\frac{6}{12}$  circle, and  $\frac{2}{8}$  rectangle) that Roo did not like off to the side. The experimenter then drew the child’s attention to the shapes on the tray with Roo and said, “See, these are all the ones that Roo likes”. See Figure 1a for an example of the final set up.

In the *Categorical* condition, all to-be-learned proportions were paired with a single nonsense word (“This is called **blick**” for  $\frac{3}{4}$  and for  $\frac{6}{8}$ ). In the *Numerator-Focused* condition, the numerator was highlighted using a number word and the denominator was obscured using a non-sense word (“This is called **three-blicks**” for  $\frac{3}{4}$  and “This is called **six-daxes**” for  $\frac{6}{8}$ ). In the *Part-Whole* condition, the denominator was first highlighted, then the numerator was

highlighted using “out-of” terminology (“Here are **four blicks** [highlighting the whole shape]. I’m going to color **three blicks**. See, this is called **three-out-of-four blicks**” for  $3/4$ , and similarly for  $6/8$ , except the pieces were labeled “daxes” instead of “blicks”). Within each condition, the use of blick(s) and dax(es) with specific fractions was counter-balanced across children.



**Figure 1:** Stimuli used during the label training (Figure 1a, left), the equivalence matching task (Figures 1b and 1c, middle), and the comparison task (Figures 1d and 1e, right). Figure 1a shows the shapes that the character (i.e., Roo) does like (upper figure, on the tray) and the shapes the character would not like (lower figure). In the center, Figure 1b shows an example of a trial without competing numerical information and Figure 1c shows a trial with competing numerical information. On the right, Figure 1d shows an example of a numerically consistent trial and Figure 1e shows an example of a numerically misleading trial.

**Learning verification.** The learning verification task tested whether children learned the specific proportions taught and were able to generalize them to novel shapes and/or equivalent proportions. Children were presented with two side-by-side stimuli on a computer on each trial, and were asked which of the shapes Roo would like. The stimuli remained on the screen until the child pointed to one of the stimuli and the experimenter recorded the child's response by pushing the corresponding keyboard key. The next trial began as soon as the experimenter recorded the child's response.

Children received six trials. The first trial included a direct comparison learned during training ( $3/4$  vs.  $3/8$ ; circles 9.8cm in diameter). The second trial included a similar comparison learned during training ( $6/8$  vs.  $4/8$ ; 13.4cm x 8.8cm rectangle with horizontal lines). The remaining four trials involved a novel shape (10cm x 10cm square divided with vertical lines) representing both learned ( $3/4$  and  $6/8$ ) and equivalent, non-learned ( $12/16$  and  $9/12$ ) proportions. Thus, the correct response was always equivalent to  $3/4$ , and proportion correct (out of 6 trials) was the dependent variable (see Appendix table A1 for full set of stimuli).

**Equivalence task.** The equivalence task was a match-to-sample task on the computer assessing whether children could match shapes based on proportional information. On each trial, children were shown a sample stimulus (circle, 7.7cm in diameter) in the upper half of the screen for 1000ms before two options (rectangles with horizontal lines, 13.4cm long x 7.7cm tall) appeared on the bottom half of the screen. A thick black line divided the upper and lower halves of the screen (see Figures 1b and 1c for example stimuli). Children were asked which of the bottom pictures "best matched" the top (sample) picture. All stimuli remained on the screen until the child gave a response and the experimenter recorded the child's response using the keyboard. The next trial began as soon as the experimenter recorded the child's response.

Children completed 13 trials (in a random order) that varied in the make-up of the options (see Appendix Table A2 for a full set of magnitudes used). Eight trials involved competing numerical information, in which one response matched on proportion but not number (i.e., was an equivalent proportion) and the other option had the same denominator (i.e., was broken up into the same number of pieces; 4 trials) or the same numerator (i.e., had the same number of yellow pieces; 4 trials) as the sample stimulus. Thus, each answer only matched on one feature (number OR proportion), making these two features in clear competition. On half of these competing numerical trials, the proportional response had a greater number of pieces (e.g., target= $2/8$ , correct answer= $4/16$ ) and on the other half the proportional response had fewer pieces (e.g., target= $2/6$ , correct answer= $1/3$ ). An additional five trials<sup>1</sup> did not involve competing numerical information: the correct answer was either an exact match (and thus, matched on number *and* proportion; 3 trials) or the correct answer was an equivalent proportion (matched on proportion, but not number) and the incorrect answer did not match the sample stimulus on either number or proportion (2 trials). The dependent variable was the number of trials (of each type: with vs. without competing numerical information) in which the child selected the proportional match.

**Comparison task** (adapted from Hurst & Cordes, 2017; Jeong et al., 2007) The comparison task assessed whether children were able to judge the relative magnitude of two probabilistic displays. Children were first introduced to a cardboard spinner (not on the computer, 8cm in diameter with a small black arrow, approximately 3cm long) and were told that

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<sup>1</sup> It is worth noting that there are a different number of trials with (8) and without (5) the opportunity for numerical interference. This was done because of the greater number of trials required for a well-balanced set of numerical interference trials. Although it's possible that this biased children's responding, this would not greatly impact the between-subject comparisons.

the yellow pieces were “winning pieces” and the grey pieces were “losing pieces”. The experimenter then spun the arrow twice and asked the child what the outcome meant.

Following this familiarization, children saw two spinners on each trial, one each on the right and left of the computer screen (see Figures 1D and 1E). Children were asked which of the spinners (on the computer) was “better”. The stimuli remained visible until the child responded and the experimenter recorded the child’s response using the corresponding keyboard keys. Spinners were circles divided into equal pieces around the center point so that numerical information was available and were presented in three different sizes to prevent direct perceptual comparisons (small circles 6cm in diameter; medium circles 8.8cm in diameter; large circles 11.5cm in diameter).

Children received 8 trials (in a random order; see Appendix Table A3 for the trial list). On half the trials, number was consistent with proportion, such that the spinner with the greater proportion of yellow also had the greater number of yellow pieces (“numerator”). On these trials, choosing the spinner with the higher number of yellow or the highest proportion of yellow would result in the same answer (e.g.,  $2/5$  vs.  $5/9$ ; “consistent trials”). On the other half of trials, number was inconsistent with proportion, such that the spinner with the greater proportion of yellow had a lower number of yellow pieces (e.g.,  $3/4$  vs.  $5/11$ ). Thus, choosing the spinner with the greater number of yellow pieces would result in an incorrect response (e.g.,  $4/9$  vs.  $2/3$ ; “misleading trials”). The primary dependent variable was proportion correct (i.e., proportion of trials selecting the more probable “yellow” spinner).

## **Results and Discussion**

Initial analyses revealed no differences across conditions on key variables, ensuring random assignment across the three conditions. Across conditions, children did not significantly

differ in terms of their age,  $F(2,122)=0.44$ ,  $p=0.64$ , *partial*  $\eta^2<0.01$  or vocabulary,

$M_{\text{Categorical}}=19.6$ ,  $M_{\text{Numerator}}=19.5$ ,  $M_{\text{Part-Whole}}=19.8$ ,  $F(2,105)=0.08$ ,  $p=0.9$ , *partial*  $\eta^2<0.01$ .

However, given the relatively large age range tested, we controlled for age in all between-subject analyses presented below. Notably, when these analyses control for vocabulary instead (on the subset of children who have valid vocabulary measures), the pattern of results is identical (the full analyses, including those controlling for vocabulary instead of age, are available at <https://osf.io/z4xhv/>).

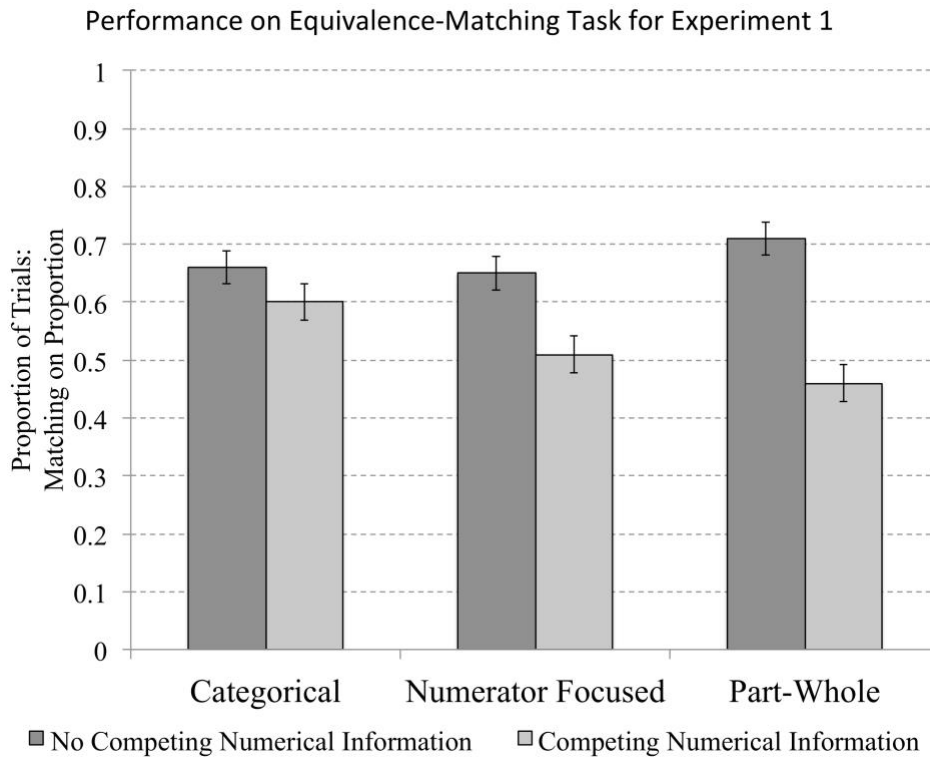
### **Learning Verification Task.**

Performance on the learning verification task was above chance in all three conditions,  $M_{\text{Categorical}}=0.79$ ,  $t(42)=10.7$ ,  $p<0.001$ ;  $M_{\text{Numerator}}=0.76$ ,  $t(40)=8.0$ ,  $p<0.001$ ;  $M_{\text{Part-Whole}}=0.74$ ,  $t(40)=7.3$ ,  $p<0.001$ . A one-way ANCOVA exploring the effect of condition, including age as a covariate, did not reveal a significant main effect:  $F(2,121)=0.72$ ,  $p=0.5$ , *partial*  $\eta^2=0.01$ . Thus, children in all three conditions were able to learn the proportions that the character liked during training and generalize this to additional equivalent proportions and novel shapes.

### **Equivalence Task.**

An ANCOVA with Numerical Interference (2: present or absent) as a repeated measure, Condition (3: Categorical, Numerator-Focused, Part-Whole) as a between-subject variable, and age as a covariate was conducted on the proportion of trials children selected the proportional match. Critically, there was a significant Condition X Numerical Interference interaction,  $F(2,121)=6.5$ ,  $p=0.002$ , *partial*  $\eta^2=0.097$  (Figure 2). Thus, we investigated the numerical interference effect within each condition and trial type comparisons across conditions.





**Figure 2:** Preference for the proportional match on the equivalence-matching task in Experiment 1 across condition and trial type. Chance responding would be at 0.5. Error Bars represent Standard Error of the mean, controlling for age as a covariate.

First, we looked at the interference effect within each condition (we used Holm’s method to control the 0.05 family wise error rate and adjusted alphas are reported accordingly). Consistent with past studies showing interference effects, there were significantly lower preferences for the proportional match on numerical interference trials than trials without competing numerical information for children in the Numerator-Focused,  $M_{\text{No interference}}=0.65$ ,  $M_{\text{interference}}=0.51$ ,  $t(40)=3.3$ ,  $p=0.002$ , Cohen’s  $d=0.5$  (test 2/3: Holm’s alpha = 0.025), and the

Part-Whole,  $M_{\text{No interference}}=0.71$ ,  $M_{\text{interference}}=0.45$ ,  $t(40)=6.3$ ,  $p<0.001$ , Cohen's  $d=0.98$  (test 1/3: Holm's  $\alpha=0.017$ ), conditions. Furthermore, children in both conditions selected the proportional match significantly more often than chance on trials without competing numerical information ( $p_s<0.001$ ), but not on trials with competing numerical information ( $p_s>0.1$ ), suggesting they attended to proportion and select the proportional match only when numerical information was not in competition. In contrast, children in the Categorical condition selected the proportional match above chance on both interference,  $p=0.001$ , and non-interference,  $p<0.001$ , trials, which were not significantly different from each other,  $M_{\text{No interference}}=0.66$ ,  $M_{\text{interference}}=0.61$ ,  $t(42)=1.5$ ,  $p=0.13$ , Cohen's  $d=0.2$  (test 3/3: Holm's  $\alpha=0.05$ ). Thus, children who heard fraction labels that included number words and labeled equivalent fractions with distinct labels attended more to number when it was available, despite attending to proportion above chance when number was not in competition with proportion. On the other hand, children who heard categorical labels that did not include number words attended to proportion significantly more than chance regardless of the availability of a numerical match, and thus were not significantly impacted by the presence of distracting number information.

As a secondary way to address this interaction, we looked at performance across the three conditions on each of the two trial types separately. Using an ANCOVA across condition with age as a covariate and performance on the trials *without* interference as the dependent variable, there was not a significant main effect of condition:  $F(2,121)=1.4$ ,  $p=0.25$ , *partial*  $\eta^2=0.02$ . However, a separate, but identical, ANCOVA with performance on trials *with* the opportunity for numerical interference as the dependent variable revealed a significant effect of condition,  $F(2,121)=5.4$ ,  $p=0.006$ , *partial*  $\eta^2=0.08$ . Post-hoc comparisons across these three conditions revealed that children in the Categorical condition selected the proportional match significantly

more than children in the Part-Whole condition,  $t(121)=3.26, p=0.001$  (test 1/3: Holm's alpha = 0.017), but not significantly differently than children in the Numerator-Focused condition,  $t(121)=1.9, p=0.06$  (test 2/3: Holm's alpha=0.025). Children in the Numerator-Focused and Part-Whole conditions did not perform significantly differently from each other,  $t(121)=1.31, p=0.19$  (test 3/3: Holm's alpha=0.05).

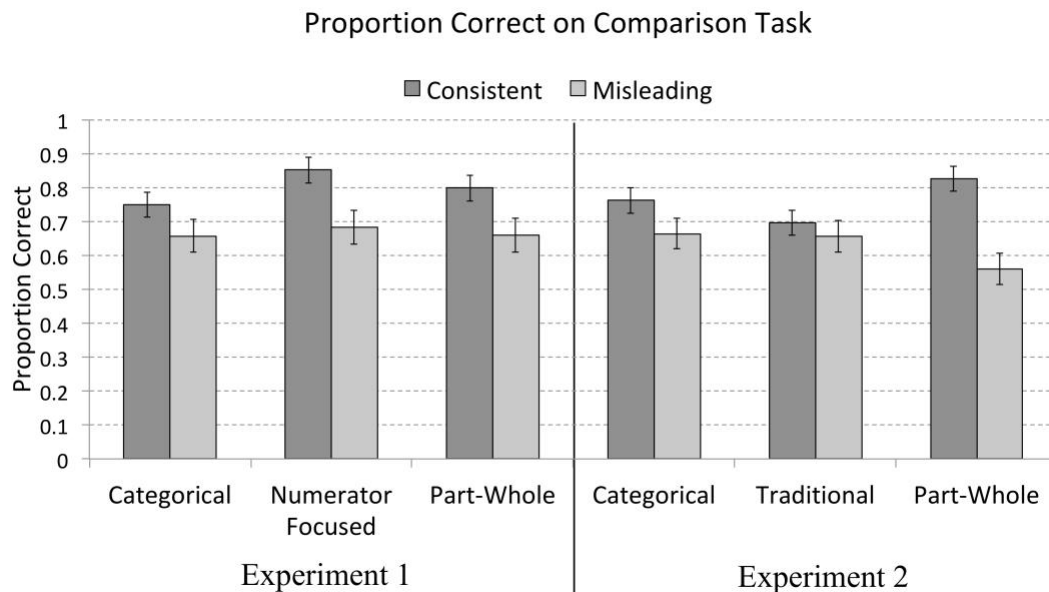
Together, consistent with our predictions, children who heard fraction labels that included number words and distinctly labeled equivalent fractions demonstrated a typical numerical interference effect, whereas children who heard categorical labels, that did not include number words, did not.

Given previous work showing a benefit of using part-whole language (Paik & Mix, 2008), it may be surprising that numerical interference was particularly high in our Part-Whole condition (although, it was not significantly different than the Numerator-Focused condition). These previous studies reporting benefits of these labels have typically involved written numeric symbols, so it is possible that the benefit of the part-whole language might be isolated to contexts involving written fraction symbols. Alternatively, however, our Part-Whole condition also included more verbal information than either of the other two conditions, in addition to a different label. Thus, it is possible that these results may have been driven by children's difficulty following our more elaborate verbal prompts. We address this in Experiment 2, by removing the extra phrasing in the Part-Whole condition and using only a modified label.

**Comparison task.** Next, we explored whether the label training with equivalent non-symbolic proportions extended to a novel context: proportion magnitude comparison. Given that numerical information was available on all trials, the critical test of numerical interference was whether performance significantly differed between trials when numerical information was

consistent with proportion relative to when it was inconsistent. That is, if children did not rely upon numerical information then performance should be relatively similar, regardless of the utility of numerical information. But if children did focus on numerical information, they should perform better when numerical information is useful (i.e., consistent with proportion) and worse when numerical information is misleading (i.e., inconsistent with proportion).

We conducted an ANCOVA on proportion correct with trial type as a repeated measure (2: Consistent v. Misleading), condition as a between-subject factor (3: Categorical, Numerator-Focused, and Part-Whole), and age as a covariate (Figure 3). There was not a significant main effect of condition,  $F(2,121)=0.95$ ,  $p=0.39$ ,  $partial \eta^2=0.02$ , or trial type,  $F(1,121)=0.30$ ,  $p=0.59$ ,  $partial \eta^2<0.01$ , nor was there a significant interaction,  $F(2,121)=0.46$ ,  $p=0.63$ ,  $partial \eta^2<0.01$ . Thus, we do not have significant evidence for overall patterns of numerical interference or that performance differed across conditions.



**Figure 3:** Proportion correct on numerically consistent and numerically misleading trials of the spinner comparison task separated by condition and experiment. Numerical Interference is the

difference in performance on numerically consistent and misleading trials. Error Bars represent Standard Error of the mean, controlling for age as a covariate.

It is worth noting that children performed significantly above chance on both the consistent and misleading trials in each of the three conditions (all comparisons to 50%,  $ps < 0.01$ ): Categorical:  $M_{\text{Consistent}}=0.74$ ,  $M_{\text{Misleading}}=0.66$ ; Numerator-Focused:  $M_{\text{Consistent}}=0.85$ ,  $M_{\text{Misleading}}=0.68$ ; Part-Whole:  $M_{\text{Consistent}}=0.80$ ,  $M_{\text{Misleading}}=0.66$ . Thus, the impact of label training found in the equivalence task did not extend to the comparison task, but instead children may have been able to attend to proportion, despite competing numerical information, in all conditions. There are several, non-mutually exclusive explanations for this pattern, which are discussed in the General Discussion.

## Experiment 2

Experiment 2 is a replication of Experiment 1 with minor changes to address three specific purposes. First, we wanted to compare performance in the key condition in Experiment 1 (the Categorical Label condition) to performance of children hearing actual English labels. Although Experiment 1 included a numerator-focused condition that, in many ways, was conceptually similar to traditional English labels, the word used for the denominator (e.g., blicks) was more obscure than the words used in traditional labels (e.g., fourths). Thus, we included a condition that used traditional fraction labels (e.g., “three-fourths”; *Traditional Label*). Second, we included a *Simplified Part-Whole* condition in which we only included the label phrase and not the extra phrase that highlighted the denominator (i.e., we removed “Here are four blicks”), making it parallel to the other conditions. This was to ensure the particularly high numerical interference in the Experiment 1 Part-Whole condition was not entirely due to the extra verbal

information. Third, we included a measure of math knowledge outside the domain of proportion to explore the relation between math ability and children's attention to numerical versus proportional information. However, the findings involving math knowledge were unclear and exploratory in nature and thus, we will not discuss these findings in the manuscript, but report a brief summary of the results in Supplemental and the full data and analyses can be found at <https://osf.io/z4xhv/>.

## Method

**Participants.** Our sample consisted of 127 children ( $M_{\text{age}}=6.4$  years, range: 5.25 years to 7.6 years,  $n_{\text{female}}=70$ ), separated into three between-subject conditions: Categorical Label ( $n=43$ ,  $M_{\text{age}}=6.51$  years, range: 5.42 years to 7.6 years,  $n_{\text{female}}=24$ ), Traditional Label ( $n=41$ ,  $M_{\text{age}}=6.49$  years, range: 5.5 years to 7.6 years,  $n_{\text{female}}=22$ ), and Simplified Part-Whole Label ( $n=43$ ,  $M_{\text{age}}=6.28$  years, range: 5.25 years to 7.5 years,  $n_{\text{female}}=24$ ). An additional 10 children participated, but were not included in any analyses because of interference from parents or other children ( $n=1$ ), an inability to complete the tasks or follow the instructions ( $n=2$ ), or experimenter ( $n=6$ ) or computer ( $n=1$ ) error. Recruitment and consenting procedure, as well as the expected demographics of the sample, are as for Experiment 1.

Some children did not have a useable vocabulary measure because of an error resulting in them not receiving the test ( $n=1$ ) or during administration ( $n=12$ ), resulting in data from 114 children in analyses involving vocabulary (Categorical:  $n=37$ ,  $M_{\text{age}}=6.44$  years, range: 5.4 years to 7.6 years,  $n_{\text{female}}=20$ ; Traditional:  $n=37$ ,  $M_{\text{age}}=6.50$  years, range: 5.5 years to 7.6 years,  $n_{\text{female}}=20$ ; Simplified Part-Whole:  $n=40$ ,  $M_{\text{age}}=6.3$  years, range: 5.25 years to 7.5 years,  $n_{\text{female}}=21$ ).

**Design.** Children were randomly assigned to one of three conditions: *Categorical* (same as Experiment 1), *Simplified Part-Whole* (similar to *Part-Whole* condition in Experiment 1), and *Traditional* (new condition). As in Experiment 1, the conditions only differed in the verbal information provided during the training phase, all other aspects of the tasks were identical. Children completed the same tasks used in Experiment 1 followed by an additional math knowledge test (discussed only in Supplementary).

**Measures.** Visual stimuli and procedures were identical to Experiment 1, except the labels used in the training conditions and the number knowledge test (discussed only in Supplementary).

**Training Phase.** In the *Categorical* condition, the to-be-learned proportion was paired with a single nonsense word (“I’m going to color blick. See, this is called **blick**” for 3/4 and 6/8), as in Experiment 1. The *Simplified Part-Whole* condition used the same label as in the *Part-Whole* condition from Experiment 1, but did not explicitly highlight the denominator (i.e., the experimenter did not say “Here are four blicks. I’m going to color three blicks”). Instead the verbal information matched the other conditions, with only a different label (“I’m going to color three-out-of-four blicks. See, this is called **three-out-of-four blicks**” for 3/4). The *Traditional* condition used formal fraction labels taught in school (“I’m going to color three-fourths. See, this is called **three-fourths**” for 3/4). For the full scripts, see Appendix.

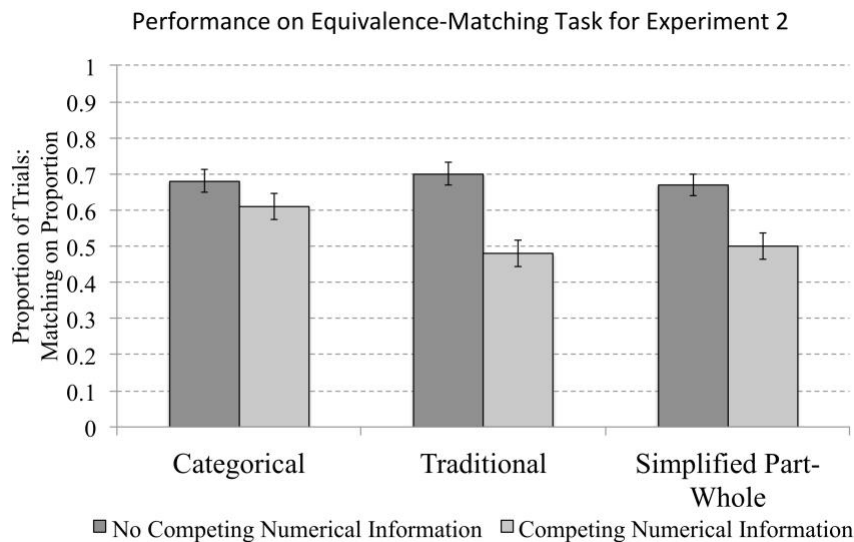
## Results and Discussion

There were no significant differences in age,  $F(2,124)=1.6$ ,  $p=0.2$ ,  $partial \eta^2=0.03$ , or vocabulary,  $M_{Categorical}=19.7$ ,  $M_{Traditional}=18.9$ ,  $M_{S. Part-Whole}=19.7$ ,  $F(2,111)=0.64$ ,  $p=0.53$ ,  $partial \eta^2=0.01$ , across conditions. As in Experiment 1, comparisons across conditions include age as a covariate, but the same result pattern is found when vocabulary is used instead (as in Experiment

1, all data and analyses, including the math test and vocabulary analyses not reported here, are available at <https://osf.io/z4xhv/>.

**Learning Verification Task.** Children in all three conditions performed above chance on the Learning Verification Task,  $M_{\text{Categorical}}=0.80$ ,  $t(42)=11.9$ ,  $p<0.001$ ;  $M_{\text{Traditional}}=0.69$ ,  $t(40)=5.1$ ,  $p<0.001$ ;  $M_{\text{S. Part-Whole}}=0.75$ ,  $t(42)=6.6$ ,  $p<0.001$ . A one-way ANCOVA across conditions, with age as a covariate, was again not statistically significant,  $F(2,123)=2.78$ ,  $p=0.07$ , *partial*  $\eta^2=0.04$ . Thus, as in Experiment 1, children in all three conditions learned the specific proportions introduced and generalize them to novel equivalent proportions and shapes.

**Equivalence matching task.** As in Experiment 1, we used an ANCOVA with Numerical Interference (2: present or absent) as a repeated measure, Condition (3: Categorical, Traditional, Simplified Part-Whole) as a between-subject variable, and age as a covariate, on proportion of trials selecting the proportional response on the equivalence task. Again, there was a significant Condition X Numerical Interference interaction,  $F(2,123)=3.28$ ,  $p=0.04$ , *partial*  $\eta^2=0.05$  (Figure 4).





**Figure 4:** Preference for the proportional match on the equivalence-matching task in Experiment 2 across condition and trial type. Chance responding would be at 0.5. Error Bars represent Standard Error of the mean, controlling for age as a covariate.

First, we looked at numerical interference in each of the three conditions separately. As expected, children showed typical numerical interference effects in both the Traditional,  $M_{\text{No interference}}=0.70$ ,  $M_{\text{Interference}}=0.48$ ,  $t(40)=4.96$ ,  $p<0.001$ , Cohen's  $d=0.78$  (test 1/3: Holm's adjusted  $\alpha=0.017$ ), and Simplified Part-Whole,  $M_{\text{No interference}}=0.66$ ,  $M_{\text{Interference}}=0.50$ ,  $t(42)=3.56$ ,  $p<0.001$ , Cohen's  $d=0.54$  (test 2/3: Holm's adjusted  $\alpha=0.025$ ) conditions. Thus, children in both conditions selected the proportional match above chance on trials that did not have the opportunity for numerical interference ( $ps<0.001$ ), but were significantly less likely to choose the proportional match, and not above chance ( $ps>0.5$ ), on trials that did provide an opportunity for numerical interference. In contrast, and mirroring Experiment 1, children in the Categorical condition performed above chance ( $ps<0.01$ ) on both trial types,  $M_{\text{No interference}}=0.68$ ,  $M_{\text{Interference}}=0.61$ , which were not significantly different from each other,  $t(42)=1.97$ ,  $p=0.06$ , Cohen's  $d=0.3$  (test 3/3: Holm's adjusted  $\alpha=0.05$ ).

As a second approach to the interaction, as in Experiment 1, we conducted two separate ANCOVAs across the three conditions on the two trial types separately, with age as a covariate. There was not a significant main effect of condition when the dependent variable was performance on the trials *without* numerical interference,  $F(2,123)=0.27$ ,  $p=0.8$ , *partial*  $\eta^2<0.01$ . In contrast, when the dependent variable was performance on trials with competing numerical information there was a significant main effect of condition,  $F(2,123)=3.58$ ,  $p=0.03$ , *partial*  $\eta^2=0.06$ . Post-hoc comparisons revealed that children in the Categorical condition selected the

proportional match more than children in the Traditional,  $t(123)=2.5, p=0.014$  (test 1/3: Holm's adjusted  $\alpha=0.017$ ), and Simplified Part-Whole,  $t(123)=2.05, p=0.04$  (although, this is not significant when controlling for the family-wise error-rate; test 2/3: Holm's adjusted  $\alpha=0.025$ ), conditions. Performance between the Traditional and Simplified Part-Whole conditions did not significantly differ,  $t(123)=0.45, p=0.6$  (test 3/3: Holm's adjusted  $\alpha=0.05$ ).

These findings replicate the pattern of Experiment 1, despite slight variations in the labels used. This suggests that the numerical interference in the Numerator-Focused and Part-Whole conditions of Experiment 1, relative to the Categorical condition, was not exclusively due to the use of nonsense words in the denominator or extra verbal information; even children who received the Traditional or Simplified Part-Whole labels in Experiment 2 showed heightened attention to numerical information when it was available, at the expense of proportion, and this pattern was significantly higher than children who heard the Categorical label.

**Comparison task.** Next, we explored performance on the comparison task (Figure 3) using an ANCOVA on proportion correct with Trial Type (2: Consistent, Misleading) as a within-subject factor, Condition (3: Categorical, Traditional, and Simplified Part-Whole) as a between-subjective factor, and Age as a covariate. There was not a main effect of Condition,  $M_{\text{Categorical}}=0.71, M_{\text{Traditional}}=0.68, M_{\text{S.Part-Whole}}=0.69; F(2,123)=0.42, p=0.66, \text{partial } \eta^2 < 0.01$ , or Trial Type,  $M_{\text{Consistent}}=0.76, M_{\text{Misleading}}=0.63; F(1,123)=0.007, p=0.93, \text{partial } \eta^2 < 0.01$ . However, there was a significant Trial Type X Condition interaction,  $F(2,123)=3.5, p=0.03, \text{partial } \eta^2=0.05$ .

To further investigate the interaction, we looked at performance in each condition separately by comparing performance on the consistent versus misleading trials in each condition using paired-samples t-tests. There was not a significant interference effect for children in the

Categorical,  $M_{\text{Consistent}}=0.77$ ,  $M_{\text{Misleading}}=0.67$ ;  $t(42)=1.6$ ,  $p=0.12$ , Cohen's  $d=0.24$  (test 2/3, Holm's  $\alpha=0.025$ ), or Traditional,  $M_{\text{Consistent}}=0.70$ ,  $M_{\text{Misleading}}=0.66$ ;  $t(40)=0.69$ ,  $p=0.5$ , Cohen's  $d = 0.1$  (test 3/3, Holm's  $\alpha=0.05$ ), conditions. Further, children in both conditions performed above chance on both trial types ( $p < 0.001$ ). However, there was a significant interference effect in the Simplified Part-Whole condition,  $M_{\text{Consistent}}=0.81$ ,  $M_{\text{Misleading}}=0.55$ ;  $t(42)=4.28$ ,  $p < 0.001$ , Cohen's  $d=0.65$  (test 1/3: Holm's adjusted  $\alpha=0.017$ ), where children performed above chance on the consistent ( $p < 0.001$ ), but not the misleading ( $p=0.3$ ) trials.

Thus, unlike Experiment 1, performance on the comparison task did significantly vary by condition. In particular, children in the Simplified Part-Whole condition showed a larger numerical interference effect than children in the Traditional and Categorical conditions. This pattern suggests that labels that emphasize equivalent proportions may also impact reasoning on a comparison task. However, given the inconsistency across the two experiments, the strength and pattern of this effect remains unclear.

### General Discussion

Across two experiments, 5- to 7-year-old children learned about a set of equivalent proportions using one of four distinct labels: a categorical label (Experiments 1 and 2), a numerator-focused label (Experiment 1), a traditional label (Experiment 2), and a part-whole label (Experiments 1 and 2). In both experiments, children who received the categorical label were more likely to select the proportional match on a subsequent matching task, despite the availability of numerical information, than children who received labels that focused on the numerator and/or the denominator (numerator-focused condition of Experiment 1 and traditional label condition of Experiment 2; complete and simplified part-whole conditions of Experiments 1 and 2). Therefore, our results indicate that the labels used to talk about proportion can have a

significant impact on directing children's attention to either proportional or numerical information.

There are at least two non-mutually exclusive explanations of these findings: (1) the categorical label promoted children's attention to proportion and (2) the labels including number words highlighted numerical information, hindering attention to proportion. The first possibility suggests that labeling multiple fractions from the same equivalence class (e.g.,  $\frac{3}{4}$  and  $\frac{6}{8}$  in this case) with the same categorical label highlighted attention to commonalities across the shapes (i.e., common proportion), allowing children to ignore those components of the shapes that differed (i.e., the number of pieces). This is in line with other work suggesting that common labels can promote categorization in infancy and childhood by highlighting similarities across differing exemplars of the same category (e.g., Fulkerson & Waxman, 2007; Graham, Namy, Gentner, and Meagher, 2010). If so, then encouraging children to think about equivalent fractions as being a category may help children attend to proportion as the category-relevant information. Although using category-based language may not be a reasonable strategy to implement in the classroom, there may be other ways to emphasize category similarities in a more sustainable and implementable way. For example, given that all equivalent fractions are equal to the same decimal value, it may be that emphasizing this translation from a single decimal to the set of equivalent fractions might also encourage categorical thinking, helping children attend to proportional magnitude.

Our second possible explanation of our findings is that the inclusion of whole-number words as part of the non-categorical labels (e.g., "three" in "three blicks") promoted attention to the *number* of pieces in each visual exemplar, at the expense of attending to proportion. As such, attending to numerical information, which varied across proportions within the same equivalence

class (i.e., “three” in  $\frac{3}{4}$  and “six” in  $\frac{6}{8}$ ), may have hindered attention to the commonality across these fractions – a common proportional magnitude. Given that we did not have a strict control group (i.e., a group that did not receive any labels) we do not have a baseline measure of how children would typically perform in this task following only the visual training. As such, we cannot distinguish between these two potential explanations with certainty. However, given other work suggesting that children around this age tend to be more inclined to match on number than proportion, even without hearing any labels (e.g., Boyer et al., 2008; Hurst & Cordes, 2017; Jeong et al., 2007), the fact that children in the categorical label condition performed more similarly across trials with and without competing numerical information than children in any of the other conditions (who were significantly less likely to attend to proportion when number and proportion were explicitly pit against each other) provides some rationale for the former. That is, the categorical label may have led children to treat the trials with competing numerical information the same way they approached the trials without competing numerical information. However, given that all conditions received some visual information about equivalent proportions, it is still an open question as to whether the number words used in the non-categorical label conditions increased children’s attention to numerical components and/or whether the single category-based label increased children’s attention to the proportional relation.

Notably, in either case the current study does suggest our current label system (which emphasizes the numerical components) may be inadvertently turning children’s attention to the specific fraction components and not toward the overall magnitude, at least for thinking about equivalent fractions. Although prior work has investigated how written symbols or labels impact children’s understanding of symbolic fractions (e.g., Ni & Zhou, 2005; Paik & Mix, 2003), the

current study is the first to suggest that the way we talk about fractions can impact children's attention to proportional versus numerical features of non-symbolic proportion well before these are formally introduced. Given the regular use of fraction words in many informal contexts (e.g., baking, money, relative time or distance), it is important to recognize that the way we talk about fractions before children learn formal symbols may be providing an early basis for the whole-number bias. Thus, future work should investigate other methods for counteracting this potentially negative influence of traditional fraction labels and encourage children to attend to the proportional amount when thinking about equivalent fractions. For example, we may be able to lower children's numerical interference by providing children with practice using continuous proportion (e.g., Boyer & Levine, 2015; Hurst & Cordes, 2017) or using decimals, which children use more easily than fractions for magnitude (Hurst & Cordes, 2018). Thus, emphasizing the continuous, equivalent nature of proportion through continuous features may help children attend to proportional information, when needed.

Importantly, we did not find that these labels solely impacted children's learning of the specific equivalence class of three-quarters. First, there was not a significant effect of condition on children's ability to learn which proportional amounts the character liked (the learning verification task), suggesting that all children were able to learn the relevant proportional category for our familiarization task. However, importantly, our matching task assessed a variety of fractions, which did not include the trained fractions, revealing that our training impacted children attention to proportion in the face of conflicting numerical information, more generally. Furthermore, children across conditions did not perform significantly differently on proportion matching trials that did not involve competing numerical information, suggesting that the different labels did not impact children's *ability* to match on proportion, but rather their strategy

or attention in particular contexts. However, one thing we cannot address with the current data is whether this pattern of findings is the result of a temporary shift of attention or whether this would have a sustained impact on children's conceptualization and learning of proportional information and formal fractions. Overall, it is not that children were better or worse able to learn a particular set of equivalent fractions or make proportional matches, but rather when we asked children to match novel proportions and contrasted these matches with competing numerical information, children who received the categorical label were more likely to match based on equivalent proportion, rather than number, while those who received number based labels were at chance responding (sometimes choosing number, sometimes choosing proportion). Further investigating the long-term relation between children's attention to number versus proportion and their learning of formal fractions is an important next step.

In contrast to other work suggesting that using part-whole labels is beneficial for children's fraction learning (Paik & Mix, 2003), in the current study we did not find the part-whole labels led to any significant differences from numerator-focused or traditional labels (Experiments 1 and 2, respectively). However, there may be several reasons for these seemingly contradictory findings. Notably, Paik and Mix (2003) focused on children's understanding of symbolic fractions, rather than proportional reasoning in the absence of symbols (as in the current study). Thus, it may be that the part-whole label structure is more apt for allowing children to understand the structure of fraction symbols, but does not make the relations between equivalent proportions any more transparent. This explanation appears reasonable, as the part-whole structure of Paik and Mix's "out-of" label directly corresponds to the structure of symbolic fractions, but does not make any reference to the proportional similarity across equivalent fractions and still uses distinct labels for equivalent proportions. Additionally,

differences in the phrasing between our part-whole condition and that of Paik and Mix (2003) were seemingly minor, but potentially important. Our part-whole condition used a nonsense word in the place of the word “parts”: “This is called three-out-of-four blicks”. On the other hand, Paik and Mix explicitly used the word parts: “of four parts, three” and “three-of-four parts”. Importantly, they found that this explicit mention of the word parts was a critical component of the label’s transparency (Paik & Mix, 2003; Mix & Paik, 2008), which may explain the continued numerical interference shown by children in our part-whole condition. Our goal was to implicitly highlight the “parts” concept by referencing the “denominator” (“look at these four blicks”) and using the phrase “out-of”, however it may be that children this young need the connection between proportional amount of pieces or parts (i.e., the unit) to be more explicit.

In addition to the equivalence-matching task, children performed a separate comparison task in which they had to judge which of two circles had a greater proportion of yellow (i.e., higher probability). In neither experiment was there a significant difference in overall performance on the numerically consistent and misleading trials, which does not provide clear evidence that children consistently made use of the numerical information, unlike other studies that have investigated this phenomenon directly (Jeong et al., 2007; Hurst & Cordes, 2018). However, it is worth noting that children in the simplified part-whole condition of Experiment 2 did show significantly higher reliance upon numerical information than children in the other two conditions. This might give some hint that the verbal labels for equivalent proportions also impacted children’s attention to proportion in the comparison task. However, the inconsistency in this finding across experiments and conditions (e.g., we would expect traditional labels in



Experiment 2 to be more like the part-whole labels than the categorical label) makes it unclear exactly what is happening in this task.

On the one hand, it may be that conceptual differences between the comparison and equivalence tasks resulted in these distinct patterns. For example, it may be that the visual information included in the training was enough to prompt children to attend to proportion in probabilistic comparison contexts, resulting in non-significant additional impacts of the verbal labels. Yet, performance on equivalence matching may not have been as impacted by the visual aspects of the training alone, allowing the labels to further impact performance. This is consistent with research suggesting that comparing probabilities may be easier and younger children may be more readily able to inhibit number based responding in these contexts than in equivalence tasks (Hurst & Cordes, 2018). On the other hand, given that this task was included at the end of the testing session it may simply be that the simple verbal training provided was not enough to have long lasting effects across multiple subtasks, especially when changing to a novel paradigm (comparison, rather than equivalence). Overall, similarities and differences between the comparison and equivalence tasks across conditions and experiments are not clear and future work should further evaluate how children approach proportional information in these distinct task contexts.

In conclusion, the current study suggests that children who are exposed to fraction labels that highlight the categorical nature of equivalent proportions are less likely to attend to irrelevant whole number information when matching equivalent proportions, focusing more on the proportional information itself. These findings highlight that the way we talk about proportion and fractions may be inadvertently turning children's attention to features that are not relevant within certain proportional reasoning contexts. Future work should continue to

investigate what factors may be guiding children's attention toward or away from the relevant features within specific concepts (e.g., comparison versus equivalence) and formats (e.g., symbolic versus non-symbolic) in order to provide a more complete picture of children's proportional reasoning.

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## Appendix

### Label Training Scripts

#### Full Script: all conditions, both experiments, except Part-Whole Experiment 1:

*“We’re going to look at some shapes that have some parts that are colored and some parts that are not colored. This is Roo. She likes when her shapes have just the right amount of color and just the right amount with no color! Even if they’re different shapes or the shapes are different sizes, the coloring amount has to be just right! Let’s see what amount of color Roo likes!”*

#### Three-Fourths:

Put the empty 4-quadrant circle on the table

*“I’m going to color [condition specific label] !”* – Color in  $\frac{3}{4}$  pieces with highlighter

*“See, this is called [condition specific label]. Roo likes this one”* – Place the colored circle on the tray next to Roo.

Put the filled in  $\frac{3}{4}$  rectangle on the table

*“This is also called [condition specific label]. Roo likes this one too!”*—Place the colored rectangle on the tray next to Roo.

Put the filled in  $\frac{3}{8}$  circle on the table: *“Roo won’t like this one”*

Put the filled in  $\frac{4}{4}$  rectangle on the table: *“Roo won’t like this one either”*

Move both of  $\frac{3}{8}$  circle (Shape 5) and the  $\frac{4}{4}$  rectangle (Shape 6) to the side, so that they’re visible but not on the tray.

#### Six-Eighths:

Put the empty 8-quadrant circle on the table

*“I’m going to color [condition specific label]!”* – Color in  $\frac{6}{8}$  pieces with your highlighter

*“See, this is called [condition specific label]. Roo likes this one”* – Place the colored circle on the tray next to Roo.

Put the filled in  $\frac{6}{8}$  rectangle on the table

*“This is also called [condition specific label]. Roo likes this one too!”* – Place the colored rectangle on the tray next to Roo.

Put the filled in  $\frac{6}{12}$  circle on the table: *“Roo won’t like this one”*

Put the filled in  $\frac{2}{8}$  rectangle on the table: *“Roo won’t like this one either”*

Move both of  $\frac{6}{12}$  circle and the  $\frac{2}{8}$  rectangle to the side, so that they’re visible but NOT on the tray.

*“Roo likes all of these!”* [With your hand, pointing to the shapes on the tray]

#### Condition Specific Labels:



Categorical (Experiment 1 and 2): *blick* OR *dax* (counterbalanced between participants)

Numerator-Focused (Experiment 1): *three-blicks* and *six-daxes* OR *three-daxes* and *six-blicks* (counterbalanced between participants)

Traditional (Experiment 2): *three-fourths* and *six-eighths*

Simplified Part-Whole (Experiment 2): three-out-of-four blicks and six-out-of-eight daxes OR three-out-of-four daxes and six-out-of-eight blicks

### **Full Script: Part-Whole Experiment 1**

Note: as above, whether “blicks” or “daxes” was paired with sixths or eighths was counterbalanced across children. Here, we are using blicks then daxes for simplicity.

*“We’re going to look at some shapes that have some parts that are colored and some parts that are not colored. This is Roo. She likes when her shapes have just the right amount of color and just the right amount with no color! Even if they’re different shapes or the shapes are different sizes, the coloring amount has to be just right! Let’s see what amount of color Roo likes!”*

Three-Fourths:

Put the empty 4-quadrant circle on the table

*“Look at these four blicks”*

*“I’m going to color three blicks”* – Color in  $\frac{3}{4}$  pieces with highlighter

*“See, this is called three out of four blicks. Roo likes this one”* – Place the colored circle on the tray next to Roo.

Put the filled in  $\frac{3}{4}$  rectangle on the table

*“This is also called three out of four blicks. Roo likes this one too!”*—Place the colored rectangle on the tray next to Roo.

Put the filled in  $\frac{3}{8}$  circle on the table: *“Roo won’t like this one”*

Put the filled in  $\frac{4}{4}$  rectangle on the table: *“Roo won’t like this one either”*

Move both of  $\frac{3}{8}$  circle (Shape 5) and the  $\frac{4}{4}$  rectangle (Shape 6) to the side, so that they’re visible but not on the tray.

Six-Eighths:

Put the empty 8-quadrant circle on the table

*“Look at these eight daxes”*

*“I’m going to color six daxes”* – Color in  $\frac{3}{4}$  pieces with highlighter

*“See, this is called six out of eight daxes. Roo likes this one”* – Place the colored circle on the tray next to Roo.

Put the filled in  $\frac{6}{8}$  rectangle on the table

*“This is also called six out of eight daxes. Roo likes this one too!”* – Place the colored rectangle on the tray next to Roo

Put the filled in  $\frac{6}{12}$  circle on the table: *“Roo won’t like this one”*

Put the filled in  $\frac{2}{8}$  rectangle on the table: *“Roo won’t like this one either”*

Move both of  $\frac{6}{12}$  circle and the  $\frac{2}{8}$  rectangle to the side, so that they’re visible but NOT on the tray.

*“Roo likes all of these!”* [With your hand, pointing to the shapes on the tray]

**Table A1:** Stimuli values for the Learning Verification Task

Correct Option	Incorrect Option
3/4	3/8
6/8	4/8
12/16	12/13
$\frac{3}{4}$	3/8
9/12	4/12
6/8	3/8

**Table A2:** Stimuli values for the Equivalence Matching Task

Trials without the opportunity for interference	Sample	Correct Match	Incorrect Match
	3/8	6/16	5/7
	2/6	1/3	4/7
	3/5	3/5	4/12
	10/18	10/18	17/18
	4/7	4/7	4/18
Trials with the opportunity for numerical interference: Denominator	Sample	Proportion Match	Number Match
	4/5	4/9	2/5
	5/7	1/4	3/7
	2/8	10/14	5/8
	6/18	8/10	10/18
Trials with the opportunity for numerical interference: Numerator	Sample	Proportion Match	Number Match
	6/7	12/14	6/13
	2/3	4/6	2/7
	4/8	2/4	3/18
	4/10	2/5	4/6

**Table A3:** Stimuli values for the Spinner Comparison Task

Trial Type	Proportion Option (i.e., correct)	Numerical Option (i.e., incorrect)
Numerically Consistent	5/7	8/9
	3/6	5/8
	2/6	5/8
	4/9	1/5
Numerically Misleading	3/5	4/9
	1/3	2/9
	5/10	4/5
	2/3	3/9